

JD on equivariant homotopy theory

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$$\begin{array}{ccc}
 K_n \times S^{n-1} & \xrightarrow{f_n} & X^{n-1} \\
 \downarrow & & \downarrow \\
 K_n \times D^n & \longrightarrow & X^n = \text{pushout}
 \end{array}$$

$K_0 = X_0$
 $K_n = \text{discrete set}$
 This is how we build
 a CW-complex
 $X = \operatorname{colim}_n X^n$

Def A G -CW cx is a CW cx as above where each K_n is a G -set and each f_n is G -equiv, where G acts trivially on S^{n-1} and D^n trivially.

Since K_n is a G -set, $K_n = \coprod_{i \in H} G/H_i$ with H_i

defined up to conjugacy. An n -dim G -cell has the form $G/H_i \times D^n$

Example $G = C_2$, $X = S^2$ with usual CW-structure and antipodal action is not a G -CW cx

This is $S(36)$ and has a G -CW structure with $K_0 \cong K_1 \cong K_2 \cong G$

Thm (Bredon) A G -map $f: X \rightarrow Y$ of G -CW cx is an equiv equiv $\Leftrightarrow f^H: X^H \rightarrow Y^H$ is an ^{ordinary} equivalence $\forall H \leq G$

Def For a G -space X , $\pi_*^H(X) := \pi_*(X^H)$

Example Let \mathcal{P} = family of proper (closed) subgps of G

Define EP , its "universal space" is the G space (unique up to equiv equiv) with

$$EP^H \simeq \begin{cases} \emptyset & \text{for } H = G \\ * & \text{else} \end{cases}$$

Construct it by attaching induced cells. See papers of Lück.

Recall A fin dim orth rep is a gp hom $\pi: G \rightarrow O(V)$ for V a fin dim Euclidean vector space.

Ex 1) Sign rep. σ

2) Regular rep ρ_G

$$\mathbb{R}[G] \text{ with } \pi(g) = \begin{pmatrix} \sum m_i g_i \\ \vdots \\ \sum_{g_i \in G} m_i g g_i \end{pmatrix}$$

3) Reduced reg rep $\bar{\rho}_G$ $V = \left\{ \sum m_i g_i : \sum m_i = 0 \right\}$

$$4) G = C_{2^n} = \langle \gamma \rangle \quad \pi(\gamma) = A = \begin{bmatrix} R_1 & & & \\ & R_2 & & \\ & & \dots & \\ & & & R_k & \pm 1 & \\ & & & & \dots & \pm 1 \end{bmatrix}$$

where $R_i = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ where $\theta = 2\pi k / 2^n$ k odd

Def say $V_1 < V_2$ if for each irred rep U ,

$$\dim \text{hom}_G(U, V_1) < \dim \text{hom}(U, V_2) - 1.$$

This means V_1 embeds in V_2 and $O(V_1, V_2)^G$ is connected.

S^V = one point compactification of V with base pt ∞ .

G fixes ∞ . representation sphere

Properties For G -reps $V, V', H \subset G, W$ a rep of H

$$1) S^{V \oplus V'} = S^V \wedge S^{V'}$$

$$2) i_H^G S^V = S^{\text{Res}_H^G V} \text{ where } i_H^G : G\text{-spaces} \rightarrow H$$

$$3) S^{\text{Ind}_H^G W} = \int^H (G_H, S^W) \text{ where } H \text{ acts on } G \text{ by left-mult}$$

$$\text{Ind}_H^G W = \mathbb{R}[G] \otimes_{\mathbb{R}[H]} W$$

G -action on $\int^H (G_H, S^W)$ is via right mult source

Ex. Let $G = C_p^n$. S^V is a G -CW complex as follows

Let $G^{(i)} =$ index p^i -subgp of G . Then

$$S^{V^{G^0}} \subset S^{V^{G^1}} \subset S^{V^{G^2}} \subset \dots \subset S^{V^{G^{(n)}}} = S^V$$

G acts trivially on $S^{V^{G^i}}$. Attach a simple $|V^{G^i}|$ -cell

To get $S^{V^{G^{(i)}}}$ from $S^{V^{G^{(i-1)}}}$ by attaching $G/G^{(i)} \times D^n$ for $|V^{G^{(i-1)}}| < n \leq |V^{G^{(i)}}|$.

$$\pi_V^H X \cong \pi_0^H \Omega^V X = [S^V, X]^H \text{ where } H \text{ is a rep of } G.$$